The +hinkingMath Handy Guide

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MISCONCEPTIONS CLARIFIED

Misconception due to misinterpretation or an inability to understand context often occurs and gives rise to errors when solving mathematical word problems, short-answer and even multiple-choice questions.

This can be frustrating when you thought that your child has fully grasped all mathematical concepts and worked tirelessly through all materials to reinforce understanding and yet he could not get the questions right. Why is this so? And what can you do about it?

This +hinkingMath Handy Guide, developed by the onSponge team, addresses the 10 most common misconceptions about word problems. Presented in an easy-to-understand format, it explains how each misconception arises and how students can avoid it. This guide aims at reinforcing the areas of improvement — with a study of the reasons and practice questions, it will help eliminate errors due to misconceptions and bring your child closer to that A/A*.

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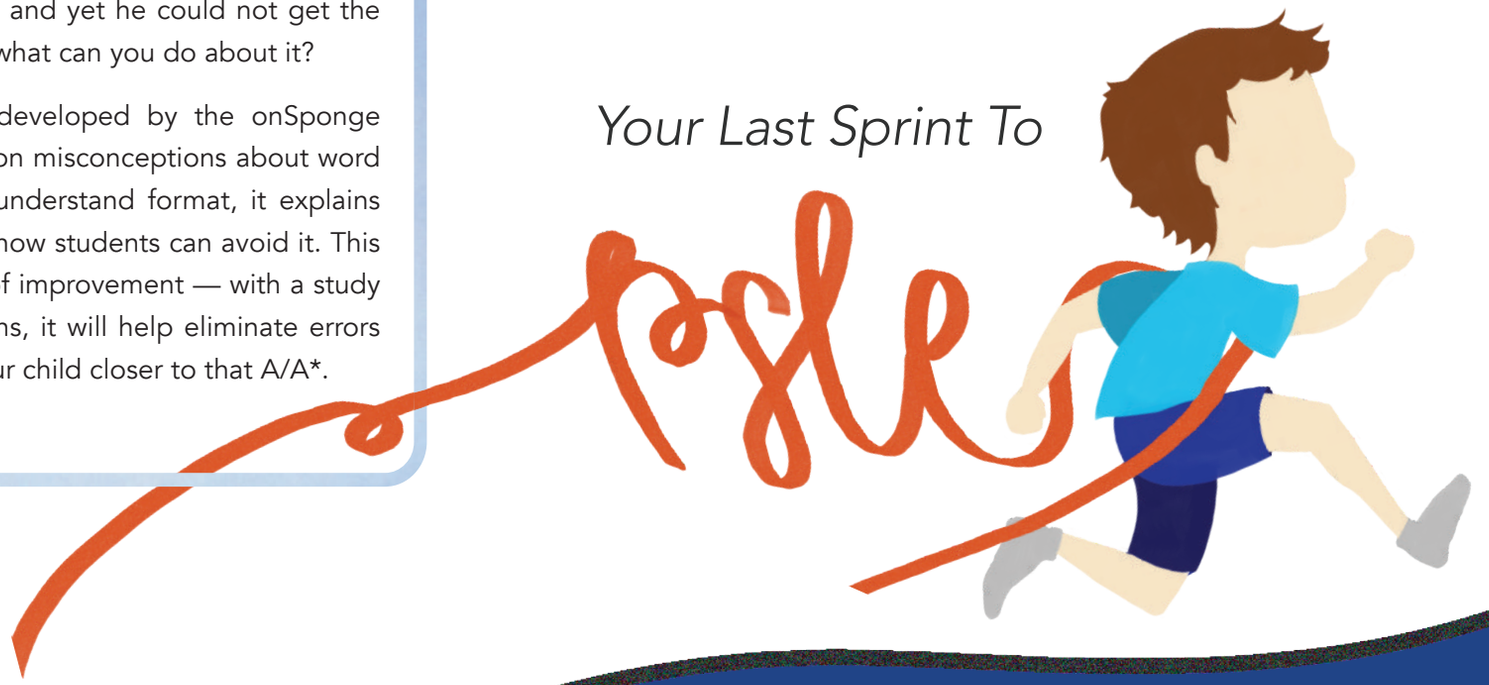
Your Last Sprint To



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Learning Resource By

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Handy Guide – Misconceptions Clarified Solutions

Solutions to Handy Guide – Misconceptions Clarified

Question 1

(a) No. of buses = 520 ÷ 12

$= 43\frac{1}{3}$

≈ 44

The least number is **44.**

(b) Total cost = 44 × 65

= 2860

Cost per student = 2860 ÷ 520

= 5.50

Each student must pay **$5.50.**

Question 2

14 m = 1400 cm

No. of strings per roll = 1400 ÷ 65

= 21 R35

≈ 21

(a) No. of rolls = 170 ÷ 21

= 8 R2

≈ 9

The least number is **9** rolls.

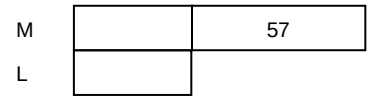
(b) Amount spent = 9 × 5

= 45

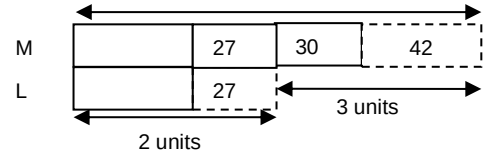
Sam needs to spend **$45** altogether.

Question 3

1st hour



After 30 min 5 units



3 units = 30 + 42

= 72

1 unit = 72 ÷ 3

= 24

7 units = 7 × 24

= 168

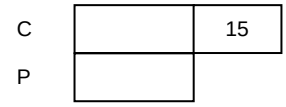
Total (1st hour) = 168 – 27 – 42

= 99

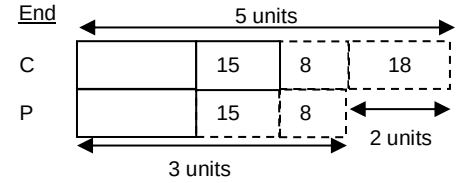
They made **99** sushi rolls in the first hour altogether.

Question 4

At first



End



2 units = 18

1 unit = 18 ÷ 2

= 9

8 units = 8 × 9

= 72

Amount raised = 72 × 0.80

= 57.60

The two boys raised **$57.60** for charity.

Question 5

Items	**Quantity**	×	**Value (erasers)**	**Total Value (erasers)**
P	1 unit + 6	×	5	5 units + 30
B	1 unit	×	8	8 units

Pencil cases Boxes

5 units + 30 = 8 units + 12 (with leftover)

8 units – 5 units = 30 – 12

3 units = 18

1 unit = 18 ÷ 3

= 6

Total erasers = 5 units + 30

= 5 × 6 + 30

= 60

There were **60** erasers altogether.

Question 6

No. of coins (At first) = 18 + 17

= 35

No. of coins (End) = 39

Total value = 39 × $0.10

= $3.90

They had **$3.90** of coins at first.

No. of 10¢	Value ($)	No. of 20¢	Value ($)	Total value ($)	Check
35	3.50	0	0	3.50	X
34	3.40	1	0.20	3.60	X
31	3.10	4	0.80	3.90	√

(+1 between the 0 and 1 rows; +0.10 between the 3.50 and 3.60 rows)

Target difference = 3.90 – 3.50

= 0.40

(b) No. of 20¢ coins = 0.40 ÷ 0.10

= 4

They had a total of **4** 20¢ coins at first.

Question 7

	First 8 coins	Next 12 coins	Subsequent 50¢ coins (Same number)
A	8 20¢ coins 8 × \$0.20 = \$1.60	12 50¢ coins 12 × \$0.50 = \$6	(6) 50¢ coins
B	8 20¢ coins 8 × \$0.20 = \$1.60	12 20¢ coins 12 × \$0.20 = \$2.40	(6) 50¢ coins
C	8 50¢ coins 8 × \$0.50 = \$4	12 50¢ coins 12 × \$0.50 = \$6	(6) 50¢ coins

(a) Most = **Cliff**, Least = **Bala**

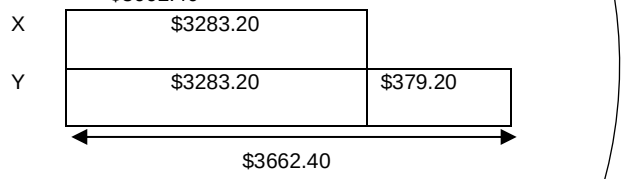
(b) Difference in total value = \$6 – \$2.40
= \$3.60
The difference in the total value between Ahmad and Bala is **\$3.60**.

(c) Difference between Bala and Cliff (At first)
= (\$4 + \$6) – (\$1.60 + \$2.40)
= \$6
Difference between Bala and Cliff (End) = \$9
Bala used = \$9 – \$6
= \$3
No. of 50¢ coins Bala used = \$3 ÷ \$0.50
= 6
No. of 50¢ coins Cliff had = 8 + 12 + 6
= 26
Cliff had **26** 50-cent coins.

Question 8

	First 24 toys	Next 30 toys	Subsequent toys (Same number)
X	24A (24 × \$60.80 = \$1459.20)	30A (30 × \$60.80 = \$1824)	(13) B
Y	24B (24 × \$76.60 = \$1838.40)	30A (30 × \$60.80 = \$1824)	(13) A

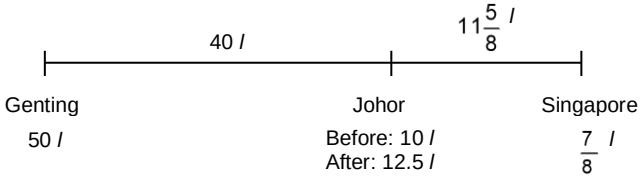
For the first 54 toys,
X earned = \$1459.20 + \$1824
= \$3283.20
For the first 54 toys,
Y earned = \$1838.40 + \$1824
= \$3662.40



Current difference
= \$3662.40 – \$379.20
= \$379.20
Target Difference
= \$379.20 – \$173.80
= \$205.40
Difference between 1A and 1B
= \$76.60 – \$60.80
= \$15.80
No. of remaining toy aeroplanes
= \$205.40 ÷ \$15.80
= 13

(a) Total Type A toy aeroplanes sold by Y
= 30 + 13
= 43
Shop Y sold **43** Type A toy aeroplanes.
(b) Total amount of money X earned
= \$3283.20 + (13 × \$76.60)
= \$4279
Shop X earned **\$4279**.

Question 9



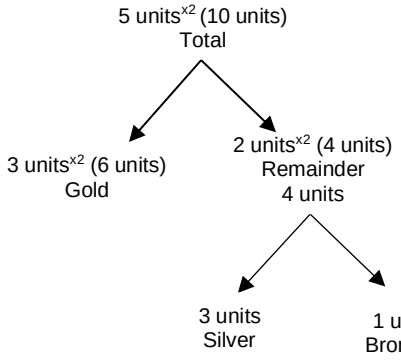
From Genting to Johor,
His car consumed = $\frac{4}{5} \times 50$ *l*
= 40 *l*
At Johor,
He pumped to = $\frac{1}{4} \times 50$ *l*
= 12.5 *l*
From Johor to Singapore,
His car consumed = 12.5 *l* – $\frac{7}{8}$ *l*
= $11\frac{5}{8}$ *l*
Total consumed = 40 *l* + $11\frac{5}{8}$ *l*
= $51\frac{5}{8}$ *l*
Wesley's car consumed $51\frac{5}{8}$ ***l*** of fuel from Genting Highlands to Singapore.

Question 10

W : M
2 : 7
Total = 2 units + 7 units
= 9 units
9 units = 630
1 unit = 630 ÷ 9
= 70
(a) W = 2 units
= 2 × 70
= 140
There were **140** women at the launch.

(b) Difference = 7 units – 2 units
= 5 units
5 units = 5 × 70
= 350
There were **350** more men than women at the launch.

Question 11



Summary
G = 6 units
S = 3 units
B = 1 unit
Total = 10 units

10 units = 800
(a) 1 unit = 800 ÷ 10
= 80
Mdm Ng had **80** bronze cards at first.

(b) B : G + S
2 : 3
G + S = 9 units
= 9 × 80
= 720
3 parts = 720
1 part = 720 ÷ 3
= 240
2 parts = 2 × 240
= 480
Bronze to buy = 480 – 80
= 400
Mdm Ng must buy **400** more bronze cards.

Question 12

(a) ∠ CAG = (180° – 74°) ÷ 2
= 53° (base ∠ of isos. triangle)
∠ FAC = 53° – 10°
= 43°
∠ FAC is **43°**.

(b) ∠ ACG = 53°
∠ ACD = 180° – 43°
= 137°
∠ ECD = 137° – 29° – 53°
= 55°
∠ ECD is **55°**.

Question 13

(a) ∠ ABG = 90° – 54°
= 36°
∠ ABG is **36°**.

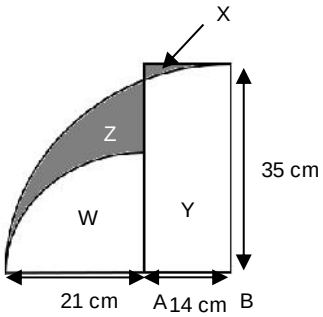
Since ABCD is a square, AB = AD.
Since AD = AE (given), AB = AD = AE.
Hence Triangle AEB is an isosceles triangle.
∠ EAB = 180° – 36° – 36°
= 108° (isos. triangle)
∠ EAD = 108° – 90°
= 18°
∠ ADE = (180° – 18°) ÷ 2
= 81° (base ∠ of isos. triangle)
∠ CDF = 180° – 81° – 90°
= 9°
(b) ∠ CFD = 180° – 9° – 90°
= 81°
∠ CFD is **81°**.

Question 14

∠ BFE = 180° – 74°
= 106°
∠ BFG = 106° – 74°
= 32°
= ∠ BGF (base ∠ of isos. triangle)
(a) ∠ FBG = 180° – 32° – 32°
= 116°
∠ FBG is **116°.**
∠ FAD = 180° – 116°
= 64° (interior ∠)
∠ HGA = ∠ BGF
= 32° (vertically opp. ∠)
(b) ∠ AHG = 180° - 64° – 32°
= 84°
= ∠ CHD (vertically opp. ∠)
∠ CHD is **84°**.

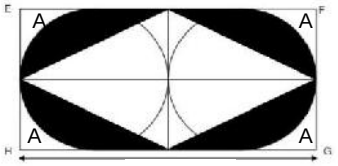
Question 15

X + Y = 14 × 35
= 490
$W = \frac{1}{4} \times \frac{22}{7} \times 21 \times 21$
= 346.5
$W + Y + Z = \frac{1}{4} \times \frac{22}{7} \times 35 \times 35$
= 962.5
Y + Z = 962.5 – 346.5
= 616
Z – X = (Y + Z) – (Y + X)
= 616 – 490
= 126
The difference between the area of the shaded regions is **126 cm²**.



Question 16

Radius of circle = 80 ÷ 4
= 20



Area of 4A = Area of square – Area of circle
= (40 × 40) – (3.14 × 20 × 20)
= 1600 – 1256
= 344
Area of 2 unshaded triangles = $2 \times (\frac{1}{2} \times 80 \times 20)$
= 1600
Area of shaded parts = (80 × 40) – 344 – 1600
= 3200 – 344 – 1600
= 1256
The area of the shaded parts is **1256 cm²**.

Question 17

Distance (1 revolution) = $\frac{22}{7}$ × 42 cm

= 132 cm

Distance between U and T = 21 cm + (5 × 132 cm)

= 681 cm

The distance between U and T in Figure 2 is **681 cm.**

Question 18

Radius = 2.9

Diameter = 2 × 2.9

= 5.8

Circumference of 1 wheel = 3.14 × 5.8

= 18.212

Length of skateboard = 59.2 + 2.9 + 2.9

= 65

Remaining length = 1120 – 65

= 1055

No. of complete revolutions = 1055 ÷ 18.212

= 57.93

≈ 57

(Note: 0.93 is not a complete revolution.)

The wheel will make **57** complete revolutions.

Question 19

Volume of 1 cuboid = 26 244 cm^3 ÷ 12

= 2187 cm^3

1u × 1u × 3u = 2187 cm^3

1u × 1u × 1u = 2187 cm^3 ÷ 3

= 729 cm^3

1u = $\sqrt[3]{729 cm^3}$

= 9 cm

Height = 4u

= 4 × 9 cm

= 36 cm

The height of the figure is **36 cm.**

Question 20

3L = 4S

1L = $\frac{4}{3}$ S

4S × 4S × (3S + $\frac{4}{3}$ S) = 14 976 cm^3

$4S \times 4S \times 4\frac{1}{3}S = 14\ 976\ cm^3$

1S × 1S × 1S = 14 976 cm^3 ÷ 4 ÷ 4 ÷ $4\frac{1}{3}$

= 216 cm^3

1S = $\sqrt[3]{216\ cm^3}$

= 6 cm

1L = $\frac{4}{3}$ × 6 cm

= 8 cm

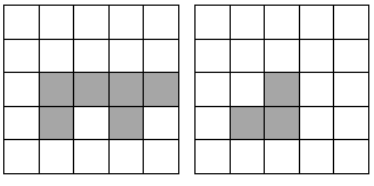
Volume of 1 small cuboid = 8 cm × 8 cm × (4 × 6 cm)

= 1536 cm^3

The volume of 1 small cuboid is **1536 cm^3.**

Question 21

Top View Side View



Question 22

(a)

Top view

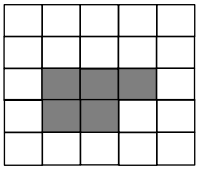


Figure 1

(b) (i) **4 cubes**

(ii) New volume = 10 × 1 cm^3

= **10 cm^3**

Question 23

Since beads P and R are aligned and, beads Q and S are aligned,

5 gaps on A	= 3 gaps on B
7 gaps on A	= 42 cm
1 gap on A	= 42 cm ÷ 7
	= 6 cm
3 gaps on B	= 5 gaps on A
	= 5 × 6 cm
	= 30 cm
1 gap on B	= 30 cm ÷ 3
	= 10 cm
Length of B	= 5 × 10 cm
	= 50 cm

The length of Wooden stick B is **50 cm**.

Question 24

7 gaps on A	= 294 cm
1 gap on A	= 294 cm ÷ 7
	= 42 cm
5 gaps on A	= 7 gaps on B
7 gaps on B	= 5 × 42 cm
	= 210 cm
1 gap on B	= 210 cm ÷ 7
	= 30 cm
Length of B	= 9 × 30 cm
	= 270 cm

1 frame → 294 cm + 270 cm = 564 cm

3 frames → 3 × 564 cm = 1692 cm

A + B + A = 564 cm + 294 cm

= 858 cm (1 roll)

B + B + A = 270 cm + 270 cm + 294 cm

= 834 cm (1 roll)

Total rolls = 1 + 1

= 2

The minimum number of rolls of ribbon Marisa needs to buy is **2** rolls.

Question 25

1M = 3B
3M = 9B
$\frac{1}{4}$ Total = 9B + 5M
$\frac{1}{4}$ Total = 3M + 5M
= 8M
$\frac{3}{4}$ Total = 3 × 8M
= 24M
Total mugs = 5 + 24
= 29
She bought **29** mugs altogether.

Question 26

$\frac{1}{4}$ Total = 10B + 5D
$\frac{3}{4}$ Total = 20D
$\frac{1}{4}$ Total = $\frac{20}{3}$ D
$\frac{20}{3}$ D = 10B + 5D
$\frac{20}{3}$ D – 5D = 10B
$1\frac{2}{3}$ D = 10B
1D = 6B
Ratio of cost of doll : cost of teddy bear (invert from qty)
6 : 1
The ratio is **6 : 1.**

Question 27

1 big rectangle = 2 small rectangles
8 big rectangles = 16 small rectangles
Total number of small rectangles = 18
Fraction = $\frac{2}{18}$
= $\frac{1}{9}$
$\frac{\mathbf{1}}{\mathbf{9}}$ of the square is covered by small rectangles.

Question 28

5 Lengths = 11 Breadths
1 Length = 11 Breadths ÷ 5
= $2\frac{1}{5}$ Breadths
Total perimeter
= 2 × (11 Breadths + 1 Length + 1 Breadth)
= 2 × (11 Breadths + $2\frac{1}{5}$ Breadths + 1 Breadth)
= $28\frac{2}{5}$ Breadths
$28\frac{2}{5}$ Breadths = 426 cm
1 Breadth = 426 cm ÷ $28\frac{2}{5}$
= 15 cm
1 Length = $2\frac{1}{5}$ Breadths
= $2\frac{1}{5}$ × 15 cm
= 33 cm
Length of Figure = 11 × 15 cm
= 165 cm
Breadth of Figure = 33 cm + 15 cm
= 48 cm
Area of Figure = 165 cm × 48 cm
= 7920 cm^2
The area of the figure is **7920 cm^2.**

Question 29

Common Difference (between even-numbered figure) = 8
Common Difference (between odd-numbered figure) = 8
General term (even-numbered figure)
= $\frac{Figure\ No.}{2}$ × 8 + 10
General term (odd-numbered figure)
= $\frac{Figure\ No. + 1}{2}$ × 8 + 7
(a) 26 + 5 = **31**

(b) 31 + 3 = **34**

(c) **8**

(d) Fig 50 → Even-numbered figure

No. of wooden planks = $\frac{50}{2}$ × 8 + 10
= 210
She would use **210** wooden planks for Figure 50.

Question 30

(a) 20 + 4 = **24**

(b) **10 cm**

(c) 1 layer → 5 cm
Fig 99 → $\frac{(99+1)}{2}$
= 50 layers
50 layers → 50 × 5
= 250
The height of Figure 99 is **250 cm**.

(d) 1st layer → 14 sticks
2nd layer → 10 sticks
3rd layer → 10 sticks
…
50th layer → 10 sticks
Since Fig 99 has 50 layers,
2nd to 50th layer → 49 layers
49 × 10 = 490
Removing Fig 100,
490 – 4 = 486
No. of sticks (Including 1st layer) = 486 + 14
= 500
She would use **500** sticks for Figure 99.

Question 31

(a) 15 + 2 = **17**

(b) **6 cm** (same as Fig 5)

(c) 1 layer → 2 cm

Fig 59 → $\frac{(59+1)}{2}$

= 30 layers

30 layers → 30 × 2

= 60

The height of Figure 59 is **60 cm**.

(d) 1st layer → 7 sticks

2nd layer → 5 sticks

3rd layer → 5 sticks

…

30th layer → 5 sticks

Since Fig 59 has 30 layers,

2nd to 30th layer → 29 layers

29 × 5 = 145

Removing Fig 60,

145 – 2 = 143

No. of sticks (Including 1st layer) = 143 + 7

= 150

She would use **150** sticks for Figure 59.
